INTERACTIONS OF MAGNETOHYDRODYNAMIC WAVES

(VZAIMODEISTVIE MAGNITOGIDRODINAMICHESKIKH VOLN)

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In this paper we consider the interactions of magnetohydrodynamic waves: fast (S^+) and slow (S^-) shocks, fast (R^+) and slow (R^-) expansion waves, and also interactions of magnetohydrodynamic waves with plane, ideally conducting, walls. The medium is assumed ideally conducting. No restrictions are imposed on the parameters of the medium.

From the moment of interaction a discontinuity arises which must be resolved into some wave combinations. In this entire paper, we determine possible wave combinations. The problem of resolving a discontinuity, which arises from the interaction of magnetohydrodynamic waves, is a particular case of the general problem of resolving arbitrary discontinuities in the magnetohydrodynamics of ideally conducting media [1].

Interactions of ordinary gasdynamical waves, as well as reflections of gasdynamical waves from plane walls, have been considered in [2,3]. Reflections from plane walls and collisions of shocks, with a magnetic field parallel to the wave fronts, have been considered in [4-6]. In this case, the problem reduces to pure gasdynamics [7]. Interactions of magnetohydrodynamic waves with Alfven and contact discontinuities has been considered in [8].

We introduce some notation. When two waves are moving one behind another to the right (left), the undisturbed state will be denoted by the index 0′ (0), the state behind the first wave by the index 1, and the state behind the second wave by the index 0 (0′). When two waves collide while moving toward each other, the undisturbed state will be denoted by the index 1, the state behind the wave moving to the right by the index 0, and that behind the wave moving to the left by the index 0′. All other notations will agree with those in [1]. For brevity, instead of the expression: "the line in the pH_y-plane, relating the quantities p and H_y on an S^+-wave (S^-, R^+, R^-), when the magnetic field and pressure in front of the wave equal H_0 and p_0", we shall write: "the S^+-line (S^-, R^+, R^-)

originating from the point (p_0, H_{y0})".

1. Collision of waves of the same type and equal intensity. Reflection of waves at a wall. We consider the case of resolving an arbitrary discontinuity, when $p_0 = p_0'$, $\mathbf{H}_{\tau 0} = \mathbf{H}_{\tau 0}'$, while Δu, Δv, Δw may be arbitrary. This case is of interest, since the problem of collision of two waves of identical type and equal intensity, and the problem of reflection of waves and Alfven discontinuities at a conducting wall, both reduce to it.

In [1], it was shown that if $p_0 \neq p_0'$, $H_{y0} \neq H_{y0}'$, then the combination consisting of two shocks or simple waves and a contact discontinuity will correspond to a point in the $\Delta u \Delta v$-plane. This result follows from the fact that such a combination is possible only for a definite intensity of diverging waves. Thus Δu and Δv will be fixed for given p_0, H_{y0}, p_0', and H_{y0}'.

In the case being considered, the intensity of the S^+-, S^--, R^+-, and R^--waves, in the combination consisting of two waves, is arbitrary. Therefore, in the $\Delta u \Delta v$-plane, this combination corresponds to a line.

Any two diverging waves, by which a discontinuity is resolved, are divided, generally speaking, by a contact discontinuity. Following the behavior of S^+-, S^--, R^+-, R^--lines in the pH_y-plane, we easily see that in the above case the two waves must be of the same type and intensity. Thus, out of two shocks or simple waves and a contact discontinuity the combinations S^+KS^+, S^-KS^-, R^+KR^+, R^-KR^- are possible.

It is easily seen that the construction of the lines, corresponding to S^+KS^+-, S^-KS^--, R^+KR^+-, R^-KR^--combinations, is reduced to relating u and v corresponding to S^+-, S^--, R^+-, and R^--waves, for which the pressure and the tangential component of the magnetic field equal p_0 and H_{y0}. Thus, for the line corresponding to the S^+KS^+-combination, using Equations (1.4) and (1.5) of [1], we have

$$u_1 = u_0 - f_+ = u_0' + f_+, \qquad \Delta u \equiv u_0 - u_0' = 2f_+$$
$$v_1 = v_0 + \varphi_+ = v_0' - \varphi_+, \quad \Delta v \equiv v_0 - v_0' = -2\varphi_+$$

Similarly, the equations for the other lines are established. The equations for the lines corresponding to S^-KS^--, R^+KR^+-, and R^-KR^--combinations are, respectively

$$\Delta u = 2f_-, \quad \Delta v = 2\varphi_-; \quad \Delta u = -2\chi_+, \quad \Delta v = 2\psi_+.$$
$$\Delta u = -2\chi_-, \quad \Delta v = -2\psi_-$$

The surfaces, to which correspond combinations of two waves and two A-discontinuities (in the above case, the A-discontinuities must be of equal strength, or else the tangential component of the magnetic field will not vanish on the contact discontinuity), are easily obtained by rotating the lines, which correspond to combinations of two waves, about the center line, as was done in [1,9].

The intersection of these surfaces with the plane $\Delta w = 0$ gives the already-constructed lines corresponding to the combinations

$$S^+KS^+, \qquad S^-KS^-, \qquad R^+KR^+, \qquad R^-KR^-$$

as well as the lines corresponding to the combinations

$$S^+AKAS^+, \qquad AS^-KS^-A, \qquad R^+AKAR^+, \qquad AR^-KR^-A$$

where the strength of the A-discontinuity equals 180°. These lines separate the regions which correspond to the combinations

$$S^+AS^-KS^-AS^+, \qquad S^+AR^-KR^-AS^+, \qquad R^+AR^-KR^-AR^+, \qquad R^+AS^-KS^-AR^+$$

For these, the intensity of the A-discontinuities must be 0 or 180°; but the surfaces of revolution separate those regions which correspond to combinations of the same form, but with rotational discontinuities of arbitrary strength.

The figure obtained coincides with Fig. 1, constructed in [9], for the study of the problem of a piston moving in a stationary medium with pressure p_0 and field $\mathbf{H}_0$.

Only in [9], the coordinates chosen were u, v, w, velocity components behind the waves and wave combinations. The lines, which in [9] correspond to relations between u and v on $Y^+ \equiv S^+$-, $Y^- \equiv S^-$-, $P^+ \equiv R^+$-, $P^- \equiv R^-$-waves, in our paper correspond to the S^+KS^+-, S^-KS^--, R^+KR^+-, R^-KR^--combinations; regions, which in [9] correspond to Y^+P^-, P^+P^-, etc. (in our notation S^+R^-, R^+R^-), in this paper correspond to $S^+R^-KR^-S^+$, $R^+R^-KR^-R^+$, etc. The vacuum lines, shown cross-hatched in Fig. 1 of [9], in this paper correspond to the resolution of an arbitrary discontinuity by the combination of two R^--waves of maximum intensity. After the passage of these waves, vacuum appears.

If $H_{y0} < 0$, $H_{y0}' < 0$, then the map will not change if the vertical axis denotes $-\Delta v$ instead of Δv. If, as before, we let the vertical axis denote Δv, then the upper part of the new map is obtained as a symmetric reflection of the lower part of the map constructed for $H_{y0} > 0$, $H_{y0}' > 0$ about the Δu-axis, and the lower part the symmetric reflection of the upper part about the same axis.

Let us apply the case of the resolution of an arbitrary discontinuity to the problem of the collision of two shocks of the same type and strength. In this case, the equations $p_0 = p_0'$; $H_{y0} = H_{y0}'$ will be exactly satisfied.

We observe that if, in the interactions of waves and discontinuities, space A-discontinuities are excluded, then the resolution problem will be a plane one. In what follows, we shall consider, for definiteness, that in the undisturbed medium $H_y > 0$. This can always be achieved by selecting the direction of the y-axis to be the direction of $\mathbf{H}_\tau$ in the undisturbed region. Then, for the collision of S^+-waves, we have a discontinuity on which $\Delta u = 2u_0 > 0$, $\Delta v = 2v_0 < 0$. This discontinuity must be resolved by a wave combination whose image in the $\Delta u \Delta v$-plane is a region lying in the quadrant $\Delta u > 0$, $\Delta v < 0$. This will be $S^+R^-R^-S^+$-, $S^+S^-S^-S^+$-, S^+S^+-combinations. If $H_{y1} < 0$, which means $H_{y0} < 0$, $H_{y0}' < 0$, then $\Delta u > 0$, and, as before, $\Delta v > 0$. But one readily sees that for $H_{y0} < 0$, $H_{y0}' < 0$ there lies in the part of the plane $\Delta u > 0$, $\Delta v > 0$ of Fig. 1 of [9] the very same region, which for $H_{y0} > 0$, $H_{y0}' > 0$, lies in the $\Delta u > 0$, $\Delta v < 0$ part of the plane.

This requires that a change in the sign of H_{y1} be followed by changes in the signs of H_{y0}, H_{y0}', and of v_0, v_0', such that the states *1* 0 and *1* 0′ are connected by shock relations. In the general case of resolving an arbitrary discontinuity, a change in the signs of H_{y0} and H_{y0}' does not lead to change in the signs of v_0 and v_0'; but the shape of the figure in the $\Delta u \Delta v$-plane changes, as was explained above. The problem is symmetric, thus the contact discontinuity is absent.

In place of an initial discontinuity, there remains a zone of stationary gas or a zone of vacuum. From symmetry, it follows that the picture of the motion from the left or from the right of this zone is the same as the result of the collision of an S^+-wave with an ideally conducting wall. Thus, for the collision of S^+-waves with ideally conducting walls, S^+R^--, S^+-, S^+S^--combinations of waves may reflect from the wall.

The form of the combination depends on the speed of the S^+-wave and the parameters of the undisturbed region between the wave and the wall. On the other hand, in the system of coordinates in which the medium on the left or on the right is stationary at infinity, we may consider the motion of the medium, from the left or from the right respectively, of the plane of the shock, as resulting from motion of a piston with a velocity equal in magnitude to that of the medium behind the colliding wave.

For the collision of S^--waves of equal intensity $\Delta u > 0$, $\Delta v > 0$. The generated discontinuity, generally speaking, may be resolved by the combinations

$$R^+S^-S^-R^+,\quad S^+S^-S^-S^+,\quad S^-S^-,\quad R^+AS^-S^-AR^+$$
$$S^+AS^-S^-AS^+,\quad AS^-S^-A,\quad S^+AR^-R^-AS^+,\quad S^+AAS^+$$

which correspond to a region lying in the part $\Delta u > 0$, $\Delta v > 0$, in the $\Delta u \Delta v$ -plane. For the collision of S^--waves with ideally conducting walls, the following combinations may reflect off the wall:

$$R^+S^-,\quad S^+S^-,\quad S^-,\quad R^+AS^-,\quad S^+AS^-,\quad AS^-,\quad S^+AR^-,\quad S^+A$$

depending on the speed of the S^--wave and the parameters of the medium between the wave and the wall.

In the interactions we have considered so far, we have not considered the part of R^+- and R^--waves. Thus, the waves diverging from both sides arise immediately after collision. Interaction of waves, in which R^+- and R^--waves participate, reduces, as in gasdynamics [2], to a process of penetration of waves, in which the flow cannot be described by simple waves. In that case, when the penetration is completed at a known time (which will be assumed in what follows), the wave combinations issuing from the penetration zone, generally speaking, will consist of shocks, simple waves, and Alfven discontinuities. In what follows, we shall determine possible combinations of waves, arising from interactions involving R^+- and R^--waves, leaving the region of penetration. This makes it possible to determine the parameters of the medium after the penetration is completed.

Let us consider the collision of two R^+-waves of equal intensity. For this $\Delta u = u_0 - u_0' < 0$, $\Delta v = v_0 - v_0' > 0$. From Fig. 1 [9] it follows that out of the interaction zone the following wave combinations will come out: $R^+AR^-R^-AR^+$, $R^+AS^-S^-AR^+$, $R^+R^-R^-R^+$, $R^+S^-S^-R^+$, $S^+AR^-R^-AS^+$, AR^-R^-A, R^+R^+, R^+AAR^+.

For the collision of an R^+-wave with an ideally conducting wall, the following wave combinations may reflect off the wall:

$$R^+AR^-,\quad R^+AS^-,\quad R^+R^-,\quad R^+S^-,\quad S^+AR^-,\quad AR^-,\quad R^+,\quad R^+A$$

For the collision of R^--waves of equal strength, $\Delta u < 0$, $\Delta v < 0$. As a result, the interaction may produce these waves: $R^+R^-R^-R^+$, $S^+R^-R^-S^+$, R^-R^-, Fig. 1 [9]. For the collision of an R^--wave with ideally conducting walls, the following waves may reflect off: R^+R^-, S^+R^-, R^-.

We observe that for the collision of one of the S^+-, S^--, R^+-, or R^--waves with another, and also for their collision with ideally conducting walls, with corresponding velocities of the interacting waves and parameters of the undisturbed medium, there may arise wave combinations containing R^--waves of maximum intensity, after the passage of which vacuum appears. From the exposition, it is clear that the solution to the piston problem in magnetohydrodynamics may be obtained as a special case of the problem of resolution of arbitrary discontinuities, taking into consideration the reflection of S^+-, S^--, R^+-, and R^--waves from ideally conducting walls.

2. Interactions of expansion waves of arbitrary intensity. Let two R^+-waves or two R^--waves interact. If H_y in the undisturbed medium is positive, then in the first case, $\Delta u < 0$, $\Delta v > 0$, and in the second, $\Delta u < 0$, $\Delta v < 0$, by Formulas (2.2) and (2.3) of [1]. For such interactions, the figures in the pH_y-plane will not be qualitatively different from Figs. 9, 10, 13, 14, of [8], obtained for the study of interactions of R^+- and R^--waves with K-discontinuities (such that the family of lines, corresponding to R^+- and R^--waves, in the pH_y-plane, depends on one parameter, while that corresponding to S^+- and S^--waves depends on two).

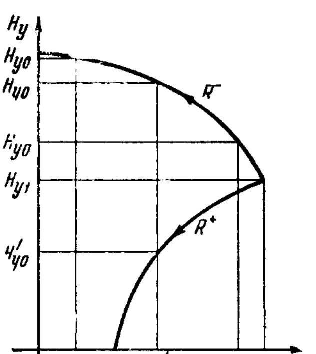

Fig. 1.

Let us consider the interaction of R^+-waves with R^--waves. For this, $\Delta u < 0$, while Δv may be positive or negative.

Let the R^+-wave collide with the R^--wave, moving to the right; for this, $H_{y0} > H_{y0}'$. (Fig. 1). Figure 1 shows the possible positions of the points (p_0, H_{y0}) and (p_0', H_{y0}').

1. If $p_0 > p_0'$ $(H_{y0}' < H_+(p_0, H_{y0}, p = p_0'))$, then in the pH_y-plane the interactions being considered correspond to Figs. 5 and 6 of [1]. After the penetration is completed, the motion will consist of combinations going to the left or to the right, which correspond to regions, lying in the $\Delta u < 0$, $\Delta v \gtrless 0$ parts of the $\Delta u \Delta v$-plane, of Fig. 9 of [1] if $p_0 < p_+(p_0', H_{y0}', H_y = H_{y0})$, or of Fig. 10 of [1] if $p_0 > p_+(p_0', H_{y0}', H_y = H_{y0})$.

2. If $p_0 < p_0'$, then in the pH_y-plane the interactions being considered correspond to Figs. 13, 14 of [1]. After the penetration is completed, the motion will consist of combinations going to the left or to the right, which correspond to regions, lying in the $\Delta u < 0$, $\Delta v \gtrless 0$ parts of the $\Delta u \Delta v$-plane, of Fig. 17 of [1] if $p_0' < p_-(p_0, H_{y0}, H_y = H_{y0}')$, or of Fig. 18 of [1] if $p_0' > p_-(p_0, H_{y0}, H_y = H_{y0}')$.

3. If $p_0 = p_0'$, then in the pH_y-plane the interactions being considered

correspond to Fig. 5 of [1]. After the penetration is completed, the motion will consist of combinations, which correspond to regions lying in the $\Delta u < 0$, $\Delta v \gtrless 0$ parts of the plane in Fig. 9 of [1].

Let an R^+-wave overtake an R^--wave, moving to the left. As before, $\Delta u < 0$ while Δv may be positive or negative.

1. If $H_{y0}' < H_{y0}$, then to these cases in the pH_y-plane correspond Figs. 11 and 12 of [1]. As a result, there will appear combinations lying in the $\Delta u < 0$, $\Delta v \gtrless 0$ parts of the $\Delta u \Delta v$-plane, of Fig. 15 of [1] if $p_0 > p_-'(p_0', H_{y0}', H_y = H_{y0})$, or of Fig. 16 of [1] if $p_0 < p_-'(p_0', H_{y0}', H_y = H_{y0})$.

2. If $H_{y0}' > H_{y0}$, then we have Figs. 3 and 4 of [1] in the pH_y-plane. As a result of the interaction, there will arise combinations, lying in the $\Delta u < 0$, $\Delta v \gtrless 0$ parts of the $\Delta u \Delta v$-plane, of Fig. 7 of [1] if $p_0 > p_+(p_0', H_{y0}', H_y = H_{y0})$, or of Fig. 8 of [1] if $p_0 < p_+(p_0', H_{y0}', H_y = H_{y0})$.

3. If $H_{y0}' = H_{y0}$, then in the pH_y-plane we have Fig. 3 of [1]. As a result of the interaction, there will appear combinations which correspond to regions lying in the $\Delta u < 0$, $\Delta v \gtrless 0$ parts of the $\Delta u \Delta v$ plane of Fig. 7 of [1].

3. Interactions of shock waves of arbitrary intensity. Let us consider interactions of shock waves with arbitrary strength. Let two S^+-waves follow one another to the right. Then, at some instant, the wave running behind (S_1^+-wave) catches up with the front wave (S_0^+-wave). Let $H_{y0}' > 0$. Then

$$H_{y1} > 0,\quad H_{y0} > 0,\quad \Delta u > 0, \Delta v < 0$$

For clarification as to which combinations will resolve the resulting discontinuity, let us turn to the corresponding curves in the pH_y-plane in Fig. 2. The point (p_0', H_{y0}') corresponds to the state of the undisturbed medium. The point (p_1, H_{y1}) on the S^+-line, issuing from the point (p_0', H_{y0}'), corresponds to the state behind the S_0^+-wave. From that point issues an S^+-line, corresponding to the $S_1'^+$-wave.

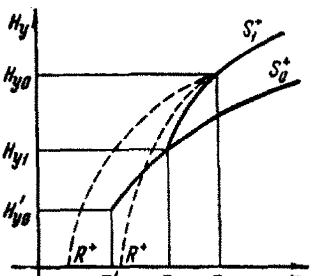

Fig. 2.

On this line lies the point (p_0, H_{y0}), where p_0 and H_{y0} are the parameters of the medium behind the S_1^+-wave. We present the following cases:

$$1.\ p_0 = p_+(p_1, H_{y1}, H_y = H_{y0}) < p_+(p_0', H_{y0}', H_y = H_{y0})$$

This indicates that the point (p_0, H_{y0}), belonging to the S_1^+-line, lies above the S_0^+-line. Then, for the resolution of the discontinuity arising from the interaction, there will appear one of the combinations which correspond to regions lying in the quadrant $\Delta u > 0$, $\Delta v < 0$ of Fig. 8 of [1] if $H_{y0}' > H_+(p_0, H_{y0}, p = p_0')$ (Fig. 4 of [1] for pH_y-plane), or of Fig. 9 of [1] if $H_{y0}' < H_+(p_0, H_{y0}, p = p_0')$ (Fig. 5 of [1] for pH_y-plane). The sign of the inequality is determined by the actual behavior of the R^+-line, issuing from the point (p_0, H_{y0}) and represented in Fig. 2 by a dotted line.

$$2.\ p_0 = p_+(p_1, H_{y1}, H_y = H_{y1}) > p_+(p_0', H_{y0}', H_y = H_{y0})$$

This indicates that the point (p_0, H_{0y}), belonging to the S_1^+-line, lies below the S_0^+-line. Then there will be realized one of the combinations which correspond to regions lying in the quadrant $\Delta u > 0$, $\Delta v < 0$ of Fig. 7 of [1] if $H_{y0}' > H_+(p_0, H_{y0}, p = p_0')$ (Fig. 3 of [1] for the pH_y-plane), or Fig. 10 of [1] if $H_{y0}' < H_+(p_0, H_{y0}, p = p_0')$ (Fig. 6 of [1] for the pH_y-plane). The sign of the inequality is determined by the behavior of the R^+-line, issuing from the point (p_0, H_{y0}).

For the collision of S^+-waves, the picture in the pH_y-plane does not change much. From the point (p_1, H_{y1}) issues an S^+-line, on which lie the points (p_0, H_{y0}) and (p_0', H_{y0}'). For definiteness, let $p_0 > p_0'$, and then $H_{y0} > H_{y0}'$. As before, $\Delta u > 0$, $\Delta v < 0$. Depending on whether the S^+-line from the point (p_0', H_{y0}') lies above or below the point (p_0, H_{y0}), we can have two different cases, as in the preceding interactions. Also as before, in each of these two cases we can have two types of resolutions, depending on the behavior of the R^+-lines issuing from the point (p_0, H_{y0}).

We now consider the interaction of S^--waves. For this, $\Delta u > 0$, $\Delta v > 0$. Considering as in the previous case the relation between p and H_y for the interaction waves, we see that two types of resolutions are possible.

$$1.\ p_0 > p_-(p_0', H_{y0}', H_y = H_{y0})$$

For the case of one rightward-moving S^--wave overtaking another, the inequality indicates that the point (p_0, H_{y0}), which lies on the S^--line issuing from the point (p_1, H_{y1}), is located above the S^--line issuing from the point (p_0', H_{y0}'). For the case of two S^--waves colliding, this indicates that the point (p_0, H_{y0}), which lies on the S^--line issuing from the point (p_1, H_{y1}), is located above the S^--line issuing from (p_0', H_{y0}').

For definiteness, we shall suppose that in the case of S^--waves colliding, the rightward-moving wave is the stronger.

Then the resulting discontinuity will be resolved by combinations, which correspond to regions, lying in the quadrant $\Delta u > 0$, $\Delta v > 0$ of Fig. 15 of [1] if $H_{y0}' < H_-(p_0, H_{y0}, p = p_0')$ (Fig. 11 of [1] for the pH_y-plane), or of Fig. 18 of [1] if $H_{y0}' > H_-(p_0, H_{y0}, p = p_0')$ (Fig. 14 of [1] for the pH_y-plane). The sign of the inequality is determined by the behavior of the R^--line issuing from the point (p_0, H_{y0}).

$$2.\ p_0 < p_-(p_0', H_{y0}', H_y = H_{y0})$$

Then the resulting discontinuity will be resolved by combinations, which correspond to regions, lying in the quadrant $\Delta u > 0$, $\Delta v > 0$ of Fig. 16 of [1] if $H_{y0} < H_-(p_0, H_{y0}, p = p_0')$ (Fig. 12 of [1] for the pH_y-plane), or of Fig. 17 of [1] if $H_{y0} > H_-(p_0, H_{y0}, p = p_0')$ (Fig. 13 of [1] for the pH_y-plane.

And here the sign of the inequality is determined by the behavior of the R^--line issuing from the point (p_0, H_{y0}).

We now consider interactions of S^+- and S^--waves. For this, either when an S^+-wave overtakes an S^--wave, or when an S^+-wave collides with an S^--wave, $\Delta u > 0$, while Δv may be either positive or negative. Consider an S^+-wave colliding with an S^--wave, moving to the left for definiteness. For this $H_{y0} > H_{y0}'$.

1. If the strengths of the interacting waves are such that $p_0 > p_0'$, then, depending on the sign of inequalities (4.1) to (4.4) of [1], i.e. depending on the behavior of the S^+- and R^+-lines issuing from the points (p_0', H_{y0}'), (p_0, H_{y0}) corresponding to Figs. 3 to 6 of [1], the generated discontinuity may be resolved by combinations which correspond to regions lying in the quadrants ($\Delta u > 0$, $\Delta v \gtrless 0$) of Figs. 7 to 11 of [1].

2. If the intensities of the interacting waves are such that $p_0 < p_0'$, then, depending on the sign of inequalities (10.1) to (10.4) of [1], i.e. depending on the behavior of the S^-- and R^--lines issuing from the points (p_0, H_{y0}), (p_0', H_{y0}') corresponding to Figs. 11 to 14 of [1], the resulting discontinuity will be resolved by combinations which correspond to regions lying in the quadrants ($\Delta u > 0$, $\Delta v \gtrless 0$) of Figs. 15 to 18 of [1].

3. If the strengths of the interacting waves are such that $p_0 = p_0'$, then the interactions being considered correspond to Fig. 3 of [1] in the pH_y-plane, and the generated discontinuity will be resolved by combinations which correspond to regions lying in the quadrants ($\Delta u > 0$, $\Delta v \gtrless 0$) of Fig. 7 of [1].

Let, now, an S^+-wave overtake an S^--wave, moving rightward for definiteness. For this, $p_0 > p_0'$.

1. If the strengths of the interacting waves are such that $H_{y0} < H_{y0}'$ (one readily sees that this occurs when $p_0 > p_-(p_0', H_{y0}', H_y = H_{y0})$, then, depending on the behavior of the R^--line issuing from the point (p_0, H_{y0}), i.e. depending on which of the inequalities (10.1), (10.4) of [1] is satisfied, the generated discontinuity is resolved by combinations which correspond to regions lying in the $\Delta u > 0$, $\Delta v \gtrless 0$ parts of the $\Delta u \Delta v$-plane of Figs. 15 and 18 of [1], respectively.

2. If $H_{y0} > H_{y0}'$, then, depending on the behavior of the S^+- and R^+-lines issuing from the points (p_0', H_{y0}'), (p_0, H_{y0}) of Figs. 3 to 6 of [1], the resulting discontinuity may be resolved by combinations which correspond to regions lying in the $\Delta u > 0$, $\Delta v \gtrless 0$ parts of the $\Delta u \Delta v$-plane in Figs. 7 to 11 of [1].

3. If $H_{y0} = H_{y0}'$, then in the pH_y-plane the interactions being considered correspond to Fig. 3 of [1] and the discontinuity arising from the interaction is resolved by combinations which correspond to regions lying in the $\Delta u > 0$, $\Delta v \gtrless 0$ parts of the $\Delta u \Delta v$-plane of Fig. 7 of [1].

4. Interaction of shock waves with expansion waves. We first recall that for the interaction of R^+-, R^--waves with S^+-, S^--waves, we shall assume that the penetration terminates after a known interval of time. After the penetration ends, depending on the strengths of the interacting waves and the parameters of the undisturbed medium, suitable combinations of waves will issue from the zone of penetration.

Let us consider the interaction of an S^+- and an R^+-wave. Let the S^+-wave overtake the R^+-wave, moving rightward. For this, Δu and Δv may be either positive or negative. Let the S^+-line issuing from the point (p_1, H_{y1}) go below (p_0', H_{y0}'). Then $p_0' < p_+(p_1, H_{y1}, H_y = H_{y0})$ (Fig. 3). In Fig. 3 are also represented the possible mutual positions of the points (p_0, H_{y0}) and (p_0', H_{y0}') in this case.

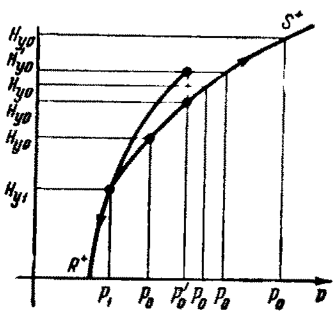

Fig. 3.

1. If $p_0 < p_0'$, then $H_{y0} < H_{y0}'$, $H_{y0} < H_+(p_0', H_{y0}', p = p_0)$. As a result of the interaction there may arise combinations which correspond to regions lying in the corresponding parts of the $\Delta u \Delta v$-plane of Fig. 10 of [1] if $p_0' < p_+(p_0, H_{y0}, H_y = H_{y0}')$ (Fig. 5 of [1] for the pH_y-plane), or of Fig. 9 of [1] if $p_0' > p_+(p_0, H_{y0}, H_y = H_{y0}')$ (Fig. 5 of [1] for the pH_y-plane). Only, the waves which are going to the right in Figs. 9 and

10 will now be going to the left, and vice versa.

2. If $p_0 = p_0'$, then $H_{y0} < H_{y0}'$. As a result of the interaction there may arise wave combinations which correspond to regions lying in corresponding parts of the $\Delta u \Delta v$-plane of Fig. 17 of [1] (Fig. 13 of [1] in the pH_y-plane).

3. If $p_0 > p_0'$, $H_{y0} < H_{y0}'$, then as a result of the interaction there may result wave combinations which correspond to regions lying in the corresponding parts of the $\Delta u \Delta v$-plane of Figs. 15 to 18 of [1] (Figs. 11 to 14 of [1] in the pH_y-plane), depending on the behavior of the R$^-$- and S$^-$-lines, issuing from the points (p_0, H_{y0}), (p_0', H_{y0}'), respectively.

4. If $p_0 > p_0'$, $H_{y0} = H_{y0}'$, then as a result of the interaction there may arise combinations which correspond to regions lying in corresponding parts of the $\Delta u \Delta v$-plane of Figs. 7 or 15 of [1] (Figs. 3 or 11 of [1] in the pH_y-plane).

5. If $p_0 > p_0'$, $H_{y0} > H_{y0}'$ (and assume for definiteness that $H_{y0}' > H_+(p_0, H_{y0}, p = p_0')$), then as a result of the interaction in this case there will appear combinations which correspond to regions lying in the corresponding parts of the $\Delta u \Delta v$-plane of Fig. 7 of [1] if $p_0 > p_+(p_0', H_{y0}', H_y = H_{y0})$ (Fig. 3 of [1] for the pH_y-plane), or of Fig. 8 of [1] if $p_0 < p_+(p_0', H_{y0}', H_y = H_{y0})$ (Fig. 4 of [1] for the pH_y-plane).

The part of the $\Delta u \Delta v$-plane in which the region corresponding to this or that combination lies, depends on the signs of Δu and Δv.

Let the S$^+$-line leaving the point (p_1, H_{y1}) go below the point (p_0', H_{y0}'). Then $p_0' > p_+(p_1, H_{y1}, H_y = H_{y0})$.

1. If $H_{y0} < H_{y0}'$, then $p_0 < p_0'$ and $H_{y0} > H_+(p_0', H_{y0}', p = p_0)$. As a result of the interaction there will arise combinations of waves which correspond to regions in Fig. 7 of [1] if $p_0' > p_+(p_0, H_{y0}, H_y = H_{y0}')$ (Fig. 3 of [1] for the pH_y-plane), and in Fig. 8 of [1] if $p_0' < p_+(p_0, H_{y0}, H_y = H_{y0}')$ (Fig. 4 for the pH_y-plane); only, wave combinations going to the right will in this case go to the left, and vice versa.

2. If $H_{y0} = H_{y0}'$, then $p_0 < p_0'$. As a result of the interaction there will arise combinations which correspond to regions situated in Figs. 7 or 15 of [1] (Figs. 3 or 11 of [1] for the pH_y-plane); again, the rightward-moving wave combinations will now go to the left, and conversely.

3. If $H_{y0} > H_{y0}'$, but $p_0 < p_0'$, then as a result of the interaction there may arise wave combinations which correspond to regions in Figs. 7 to 10 of [1] (Figs. 3 to 6 of [1] for the pH_y-plane).

4. If $H_{y0} > H_{y0}'$, $p_0 = p_0'$, then as a result of the interactions there may arise combinations of waves which correspond to the regions in Fig. 9 of [1] (Fig. 5 of [1] for the pH_y-plane).

5. If $H_{y0} > H_{y0}'$, $p_0 > p_0'$, then as a result of the interactions there may arise wave combinations which correspond to the regions in Figs. 7 to 10 of [1] (Figs. 3 to 6 of [1] in the pH_y-plane).

In the case of collision of S^+- and R^+-waves, there may result wave combinations which correspond to the regions in Figs. 7 to 10 of [1] (Figs. 3 to 6 of [1] for the pH_y-plane).

We now consider the case of S^+-waves overtaking R^--waves moving to the right. For this, $H_{y0} > H_{y0}'$, $\Delta v < 0$, and Δu may be either positive or negative. Depending on the speeds of the interacting waves and the parameters of the undisturbed medium, different wave combinations may come out.

1. If $p_0 < p_0'$ (for this case $H_{y0} \lessgtr H_+(p_0', H_{y0}', p = p_0)$), then as a result of the interactions there may arise wave combinations which correspond to the regions lying in the $\Delta u \gtrless 0$, $\Delta v < 0$ parts of the $\Delta u \Delta v$-plane of Fig. 17 of [1] if $p_0' < p_-(p_0, H_{y0}, H_y = H_{y0}')$ (Fig. 13 of [1] for the pH_y-plane), and Fig. 18 of [1] (Fig. 14 of [1] for the pH_y-plane) if $p_0' > p_-(p_0, H_{y0}, H_y = H_{y0}')$. Only, the combinations going to the left in Figs. 17 and 18 will now go to the right, and conversely.

2. If $p_0 = p_0'$, then as a result of the interaction there may arise combinations which correspond to regions lying in the $\Delta u \gtrless 0$, $\Delta v < 0$ parts of the $\Delta u \Delta v$-plane of Fig. 9 of [1] (Fig. 5 of [1] for the pH_y-plane).

3. If $p_0 > p_0'$, then as a result of the interactions there may arise combinations which correspond to regions lying in the $\Delta u \gtrless 0$, $\Delta v < 0$ parts of the $\Delta u \Delta v$-plane in Figs. 7 to 10 of [1] (Figs. 3 to 6 of [1] for the pH_y-plane).

Let now S^+-waves collide with R^--waves, moving leftward. For this, $p_0' < p_0$. As before, $\Delta v < 0$, while Δu may be positive or negative. Depending on the strength of the interacting waves, and the parameters of the undisturbed medium, different wave combinations may come out of the zone penetration.

1. If $H_{y0} < H_{y0}'$ (for this $H_{y0}' < H_-(p_0, H_{y0}, p = p_0)$), then as a result there may arise wave combinations which correspond to regions lying in the $\Delta u \gtrless 0$, $\Delta v < 0$ parts of the $\Delta u \Delta v$-plane of Fig. 15 of [1] if $p_0 > p_-(p_0', H_{y0}', H_y = H_{y0})$, or of Fig. 16 of [1] if $p_0 < p_-(p_0', H_{y0}', H_y = H_{y0})$.

2. If $H_{y0} = H_{y0}'$, then as a result of the interaction there may arise combinations which correspond to regions lying in the $\Delta u \gtrless 0$, $\Delta v < 0$ parts of the $\Delta u \Delta v$-plane of Fig. 7 of [1].

3. If $H_{y0} > H_{y0}'$, then as a result of the interaction there may arise combinations which correspond to the regions lying in the $\Delta u \gtrless 0$, $\Delta u < 0$ parts of the $\Delta u \Delta v$-plane in Figs. 7 to 10 of [1].

Let S^--waves collide with R^--waves, moving leftward. For this Δu may be either positive or negative, and $\Delta v > 0$ and $p_0' < p_0$.

Depending on the intensity of the interacting waves and the parameters of the undisturbed medium, different wave combinations may come out of the zone of penetration.

1. If $H_{y0}' > H_{y0}$, then as a result of the interaction there may arise combinations which correspond to the regions lying in the corresponding parts of the $\Delta u \Delta v$-plane in Figs. 15 to 18 of [1] (Figs. 11 to 14 of [1] for the pH_y-plane).

2. If $H_{y0}' = H_{y0}$, then as a result of the interaction there may arise combinations which correspond to regions lying in the corresponding parts of the $\Delta u \Delta v$-plane of Fig. 7 of [1] (Fig. 3 of [1] for the pH_y-plane).

3. If $H_{y0}' < H_{y0}$ (for which $H_{y0} > H_+(p_0, H_{y0}, p = p_0')$), then as a result of the interaction there arise combinations lying in the corresponding parts of the $\Delta u \Delta v$-plane of Fig. 7 of [1] if $p_0 > p_+(p_0', H_{y0}', H_y = H_{y0})$ (Fig. 3 of [1] for the pH_y-plane), or of Fig. 8 of [1] (Fig. 4 of [1] for the pH_y-plane) if $p_0 < p_+(p_0', H_{y0}', H_y = H_{y0})$.

Let R^+-waves overtake S^--waves, moving rightward. For this, $H_{y0} > H_{y0}'$; $p_0 < p_-(p_0', H_{y0}', H_y = H_{y0})$; $H_{y0} < H_+(p_0', H_{y0}', p = p_0)$.

Depending on the strengths of the interacting waves and the parameters of the undisturbed medium, different wave combinations may come out of the penetration zone.

1. If $p_0 > p_0'$, then as a result of the interaction there may be combinations which correspond to the regions lying in the corresponding parts of the $\Delta u \Delta v$-plane of Fig. 16 of [1] if $H_{y0}' < H_-(p_0, H_{y0}, p = p_0')$ (Fig. 12 of [1] for the pH_y-plane), or of Fig. 17 of [1] if $H_{y0}' > H_-(p_0, H_{y0}, p = p_0')$ (Fig. 13 of [1] for the pH_y-plane).

2. If $p_0 = p_0'$, then as a result of the interactions there may be combinations lying in the corresponding parts of the $\Delta u \Delta v$-plane of Fig. 17 of [1] (Fig. 13 of [1] in the pH_y-plane).

3. If $p_0 < p_0'$, then as a result of the interaction there may be

combinations lying in the corresponding parts of the $\Delta u \Delta v$-plane of Fig. 10 of [1] (Fig. 6 of [1] for the pH_y-plane) if $p_0' > p_+(p_0, H_{y0}, H_y = H_{y0}')$, or of Fig. 9 of [1] (Fig. 5 of [1] for the pH_y-plane) if $p_0' < p_+(p_0, H_{y0}, H_y = H_{y0}')$. Only, the waves moving leftward in these regions will move rightward, and vice versa.

Let S^--waves collide with R^--waves, moving leftward. For this Δu and Δv may be positive or negative.

As a result, there may arise wave combinations which correspond to the regions lying in the corresponding parts of the plane of Figs. 15 to 18 of [1] (Figs. 11 to 14 of [1] for the pH_y-plane), depending on the signs of Δu and Δv. Let an S^--wave overtake an R^--wave, moving leftward. Depending on the strengths of the interacting waves and the parameters of the undisturbed medium, different waves may come out from the zone of penetration. Let the S^--line in the pH_y-plane go below the R^--line.

1. If $H_{y0}' > H_{y0}$ (for this $p_0' < p_0$, $H_{y0}' < H_-(p_0, H_{y0}, p = p_0')$), then as a result of the interaction there may be combinations which correspond to regions lying in the corresponding parts of the $\Delta u \Delta v$-plane of Fig. 15 of [1] if $p_0 > p_-(p_0', H_{y0}', H_y = H_{y0})$ (Fig. 11 of [1] for the pH_y-plane, or of Fig. 16 of [1] if $p_0 < p_-(p_0', H_{y0}', H_y = H_{y0})$ (Fig. 12 of [1] for the pH_y-plane).

2. If $H_{y0} = H_{y0}'$ ($p_0' < p_0$), then as a result of the interaction there may be combinations which correspond to regions lying in the corresponding parts of the $\Delta u \Delta v$-plane of Figs. 7 or 15 of [1] (Figs. 3 or 11 of [1] in the pH_y-plane).

3. If $H_{y0}' < H_{y0}$, $p_0' < p_0$, then as a result of the interaction there may be combinations which correspond to regions lying in the corresponding parts of the $\Delta u \Delta v$-plane in Figs. 7 to 10 of [1] (Figs. 3 to 6 of [1] for the pH_y-plane).

4. If $H_{y0}' < H_{y0}$, $p_0 = p_0'$, then as a result of the interaction there may be combinations which correspond to regions lying in the corresponding parts of the plane in Fig. 9 of [1] (Fig. 5 of [1] for the pH_y-plane).

5. If $H_{y0}' < H_{y0}$, $p_0' > p_0$, then as a result of the interaction there may be combinations which correspond to regions lying in the corresponding parts of the $\Delta u \Delta v$-plane in Figs. 15 to 18 of [1] (Figs. 11 to 14 of [1] for the pH_y-plane), only, the wave combinations going leftward in this figure will now go rightward, and conversely.

Let the S^--line in the pH_y-plane go above the R^--line.

1. If $H_{y0}' > H_{y0}$ (for this $p_0' < p_0$, $H_{y0}' > H_-(p_0, H_{y0}, p = p_0')$), then as a result of the interaction there may be waves which correspond to regions lying in the corresponding parts of the $\Delta u \Delta v$-plane of Fig. 17 of [1] if $p_0 < p_-(p_0', H_{y0}', H_y = H_{y0})$ (Fig. 13 of [1] for the pH_y-plane), or of Fig. 18 of [1] if $p_0 > p_-(p_0', H_{y0}', H_y = H_{y0})$ (Fig. 14 of [1] for the pH_y-plane).

2. If $H_{y0}' > H_{y0}$, $p_0' = p_0$, then as a result of the interaction there may be combinations which correspond to regions lying in the corresponding parts of the $\Delta u \Delta v$-plane of Fig. 17 of [1] (Fig. 13 of [1] for the pH_y-plane).

3. If $H_{y0}' > H_{y0}$, $p_0' > p_0$, then as a result of the interaction there may be combinations which correspond to regions lying in the corresponding parts of the $\Delta u \Delta v$-plane of Figs. 7 to 10 of [1] (Figs. 3 to 6 of [1] for the pH_y-plane), only, wave combinations moving to the right in the figure will move to the left in this, and conversely.

4. If $H_{y0}' = H_{y0}$, $p_0' > p_0$, then as a result of the interaction there may be combinations which correspond to regions lying in the corresponding parts of the $\Delta u \Delta v$-plane of Fig. 15 of [1] (Fig. 11 of [1] for the pH_y-plane), only, wave combinations moving leftward in the region will move rightward in this case, and conversely.

5. If $H_{y0}' < H_{y0}$, $p_0' > p_0$, then as a result of the interaction there may be combinations which correspond to regions lying in the corresponding parts of the $\Delta u \Delta v$-plane of Figs. 15 to 18 of [1] (Figs. 11 to 14 of [1] for the pH_y-plane), only, wave combinations moving leftward in the figure will move rightward in this case, and conversely.

If in the pH_y-plane the behavior of the S^+-, S^--, R^+-, R^--lines is different from that considered (for example, during the interaction of S^-- and R^--waves, the S^--line intersects the R^--line, etc.), then the study of the interactions can be carried through in a manner similar to that done above. The behavior of S^+-, S^--, R^+-, R^--lines in the pH_y-plane in the general case will not be examined.

The case for the interaction of magnetohydrodynamic waves, for which R^+-, R^--waves overtake corresponding S^+-, S^--waves, is considered in a completely similar way as interactions in which S^+-, S^--waves overtake corresponding R^+-, R^--waves.

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